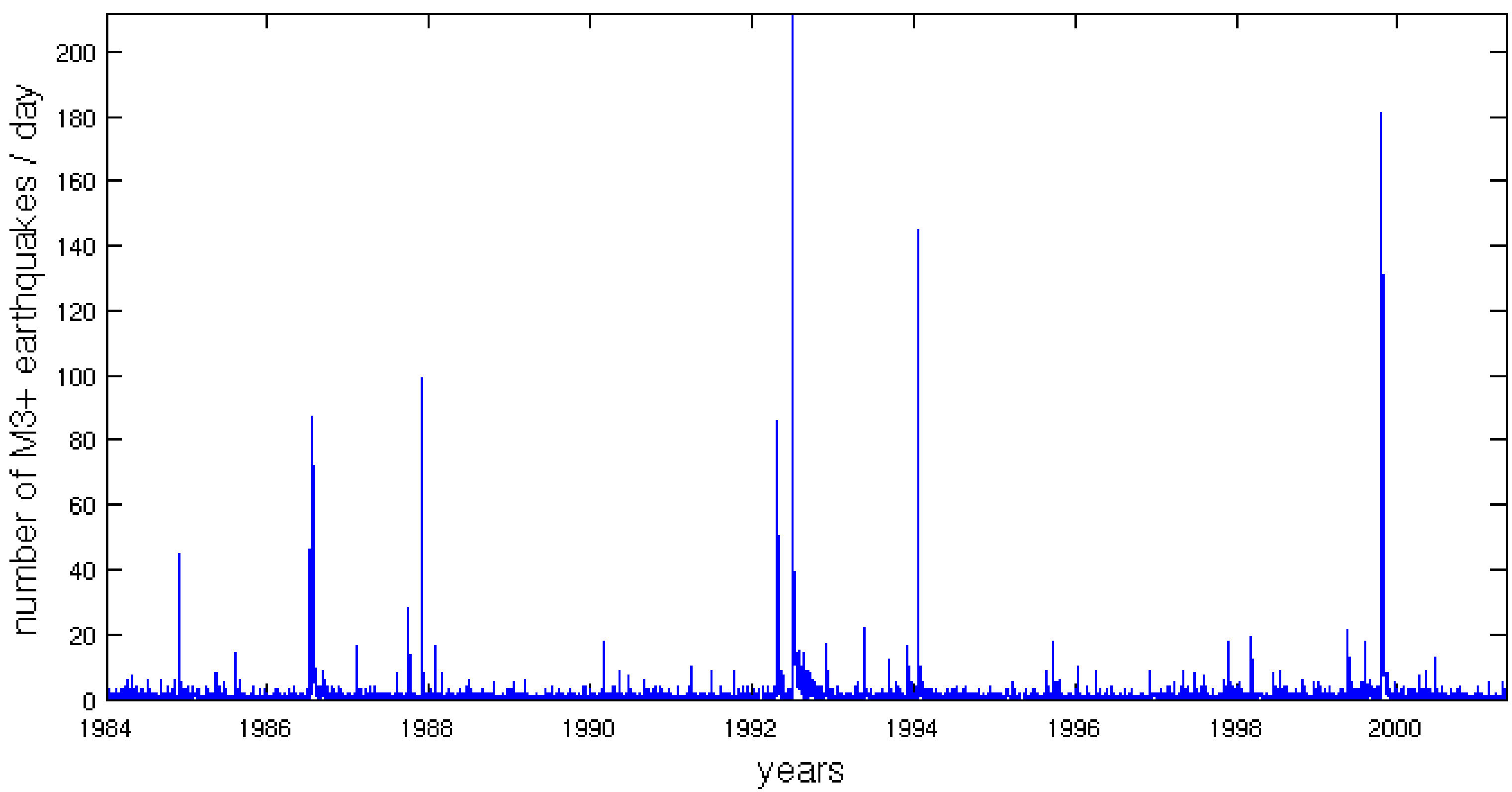
Model – Independent Stochastic Declustering

David Marsan

Laboratoire de Géophysique Interne et Tectonophysique (LGIT)
Université de Savoie
Le Bourget-du-Lac, France



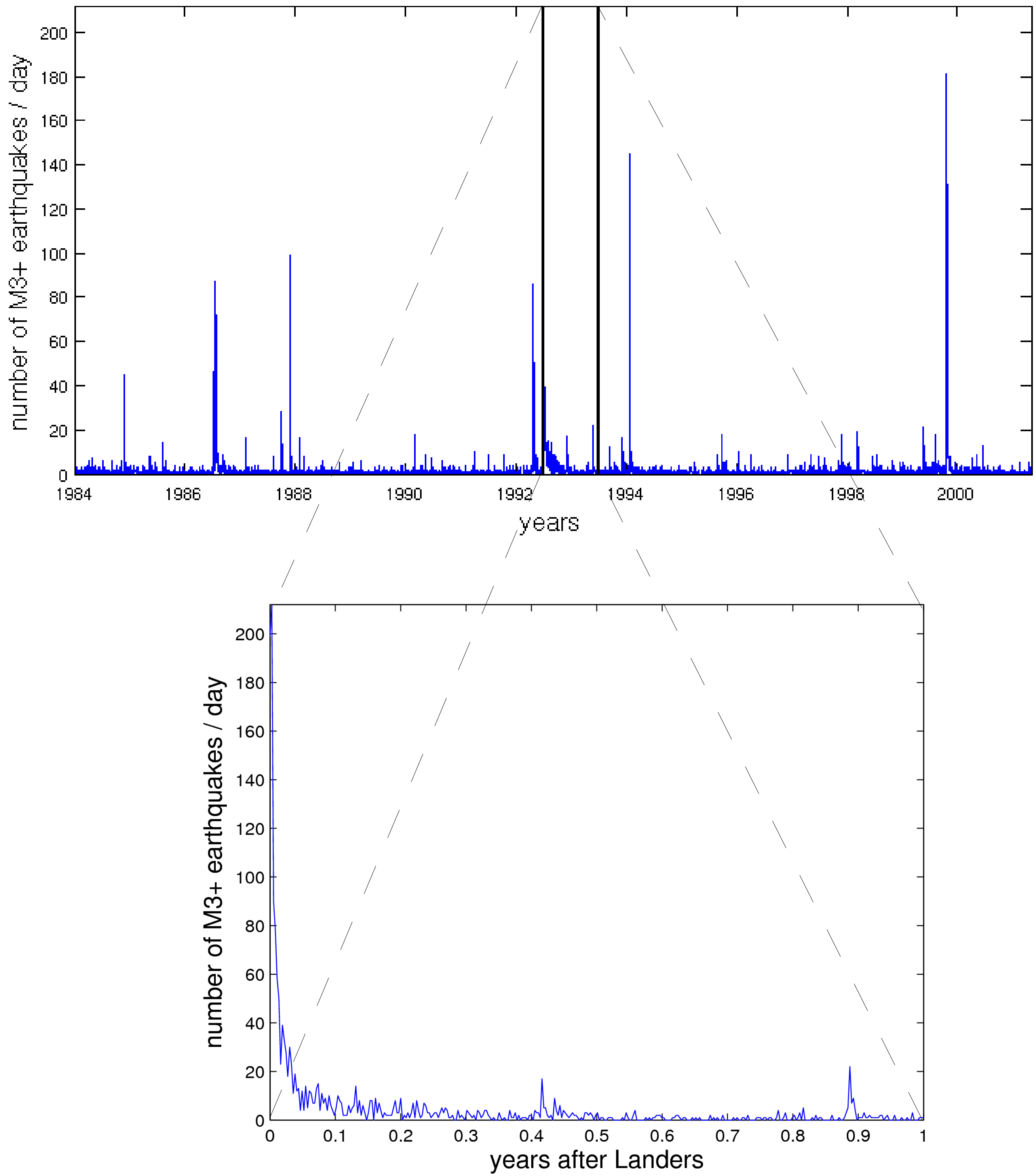
Clustering (correlation) in time and space ...

... caused by mechanical interactions in fault zones.

Questions:

(1) Which earthquake triggered what?

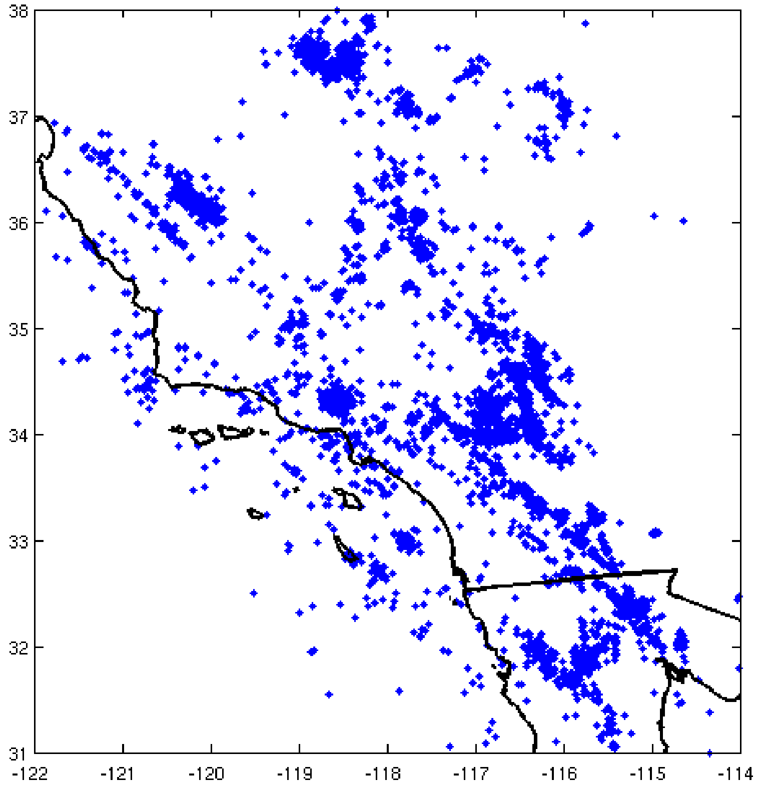
(2) How to model this triggering?

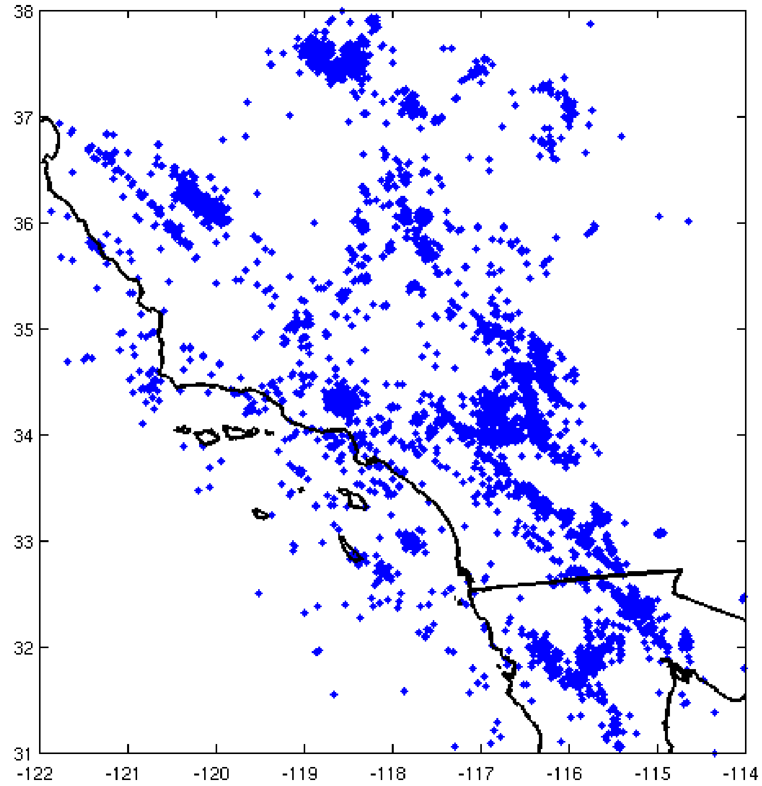


number of M3+ earthquakes / day
200
180
160
140
120
100
80
60
40
20
0
1984
1986
1988
1990
1992
1994
1996
1998
2000
years
number of M3+ earthquakes / day
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
years after Landers

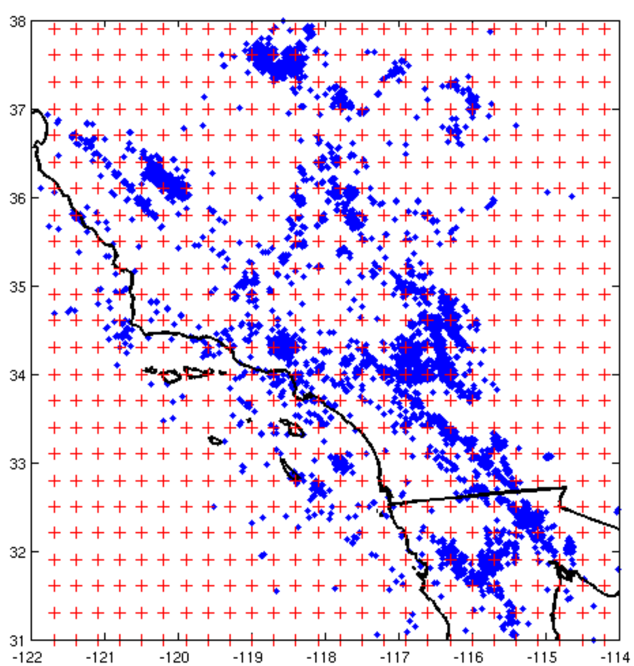
However:

(1) small earthquakes contribute heavily to stress transfer...

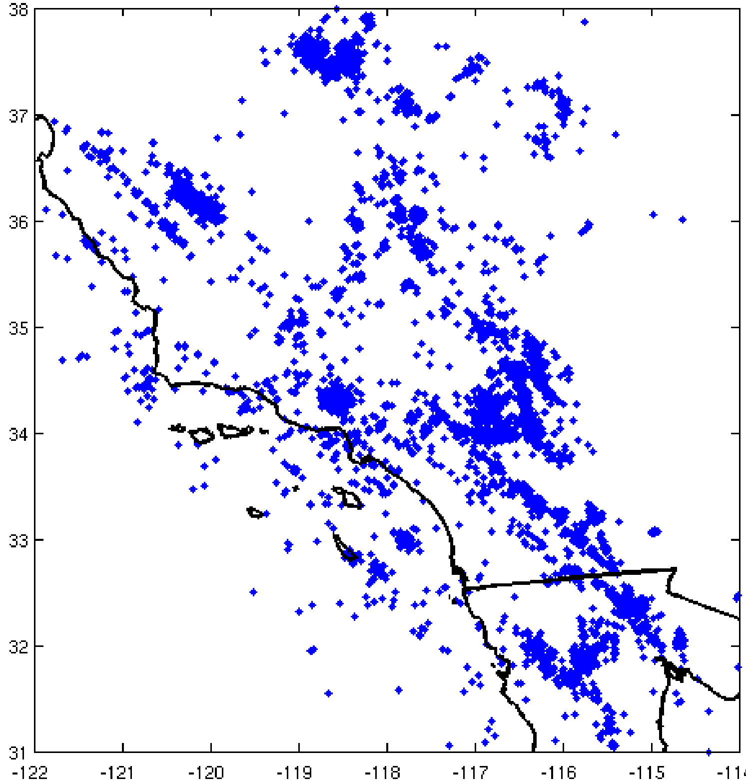

38
37
36
35
34
33
32
31
-122
-121
-120
-119
-118
-117
-116
-115
-114



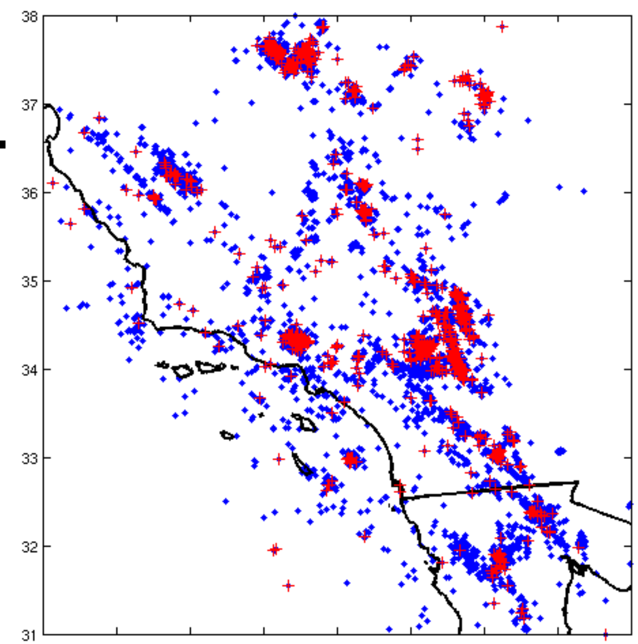
Compute stress on a regular grid



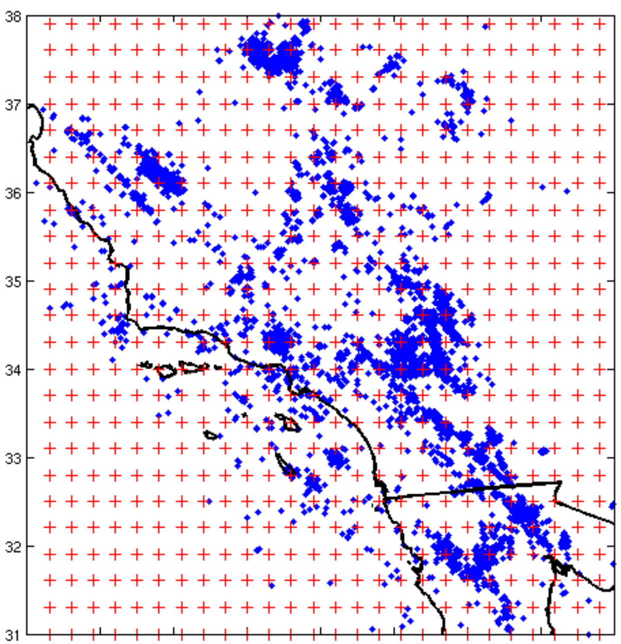
Largest earthquakes dominate



Compute stress on hypocenters



All earthquakes contribute



Marsan (GJI 2004)

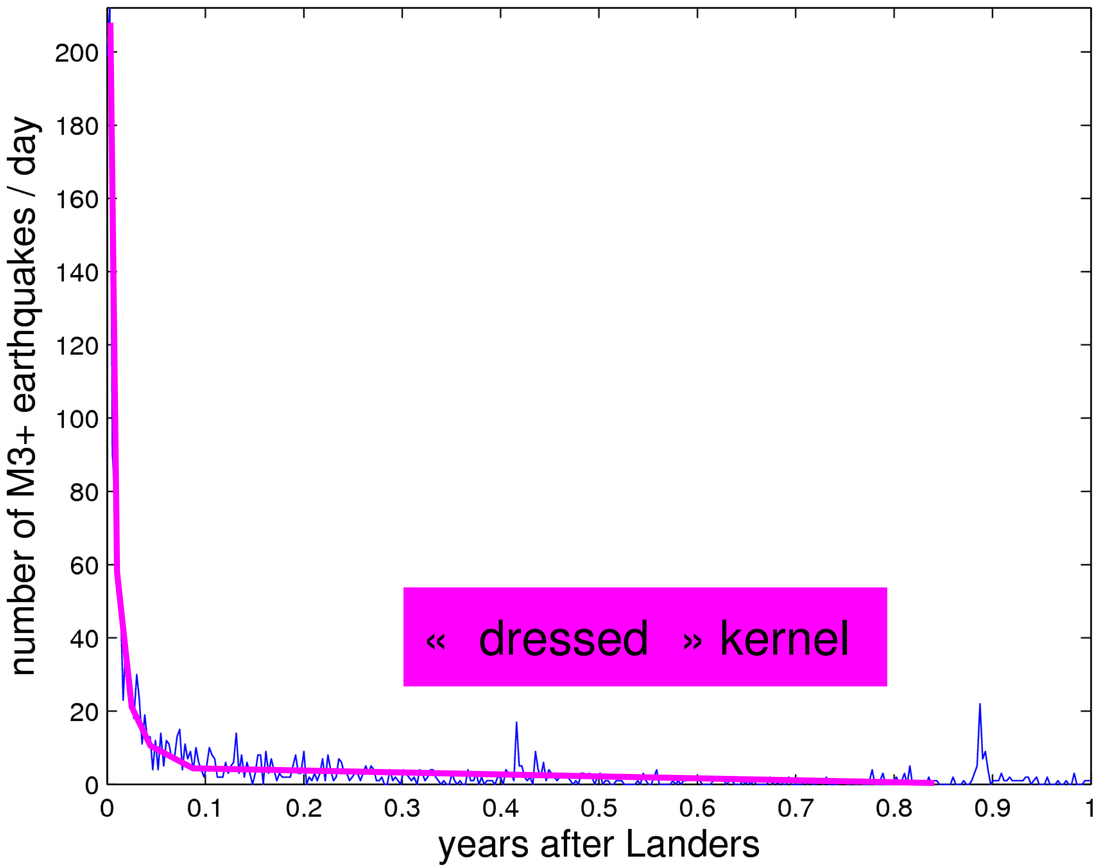
However:

(1) small earthquakes contribute heavily to stress transfer...

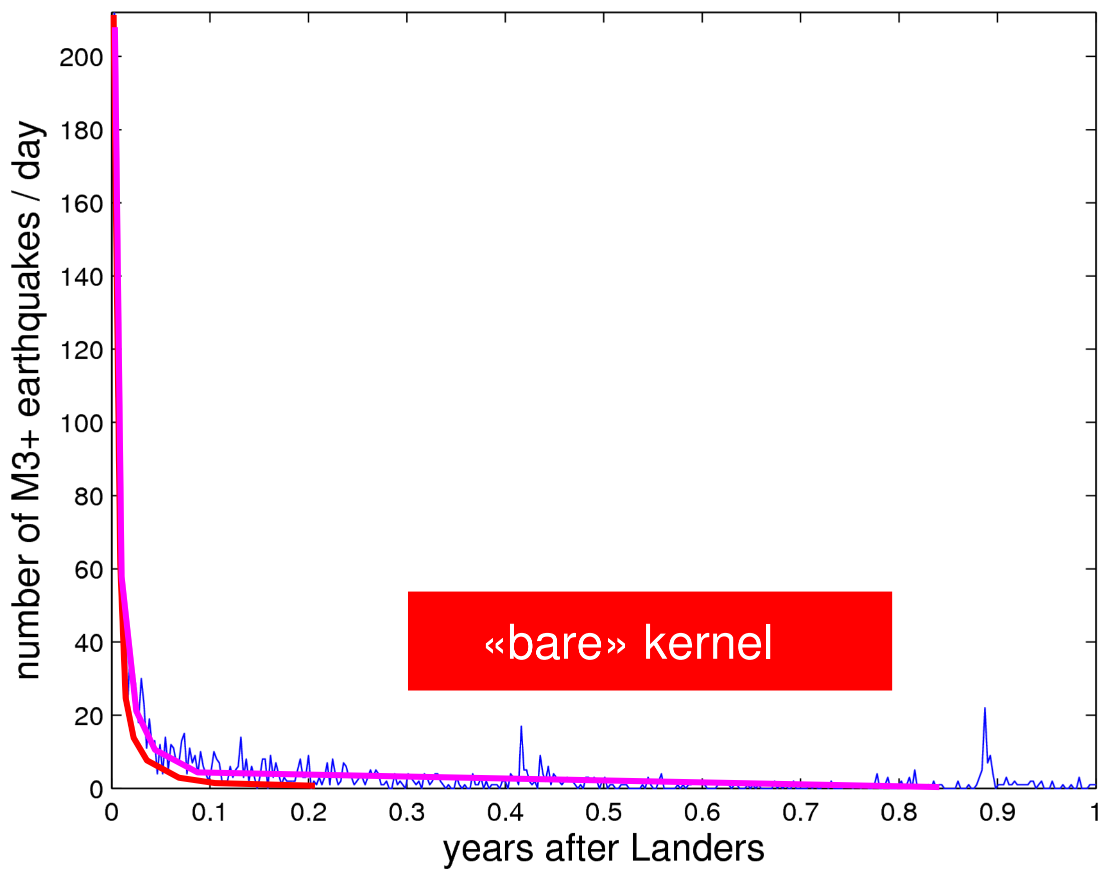
(2) ... and there exists a cascade of aftershocks.

The total triggering **(« dressed » triggering)** of aftershocks following a mainshock is the sum of:

- the triggering directly caused by the mainshock itself **(« bare » triggering)**
- the triggering caused by aftershocks of the mainshock



number of M3+ earthquakes / day
0
20
40
60
80
100
120
140
160
180
200
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
years after Landers
« dressed » kernel



number of M3+ earthquakes / day
200
180
160
140
120
100
80
60
40
20
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
years after Landers
«bare» kernel

Model – Independent Stochastic Declustering

- Estimate the various contributions (« bare » kernels) in the seismicity time series.

- Estimate a « background » forcing (Poisson process).

- Can be used to decluster earthquake datasets.

Only three assumptions:

- (1) **linearity:** the (« bare ») influences of different earthquakes sum up.

- (2) **mean field:** there exists a mean field influence (all earthquakes with the same magnitude are described with the same influence).

- (3) **background:** there exists a background rate.

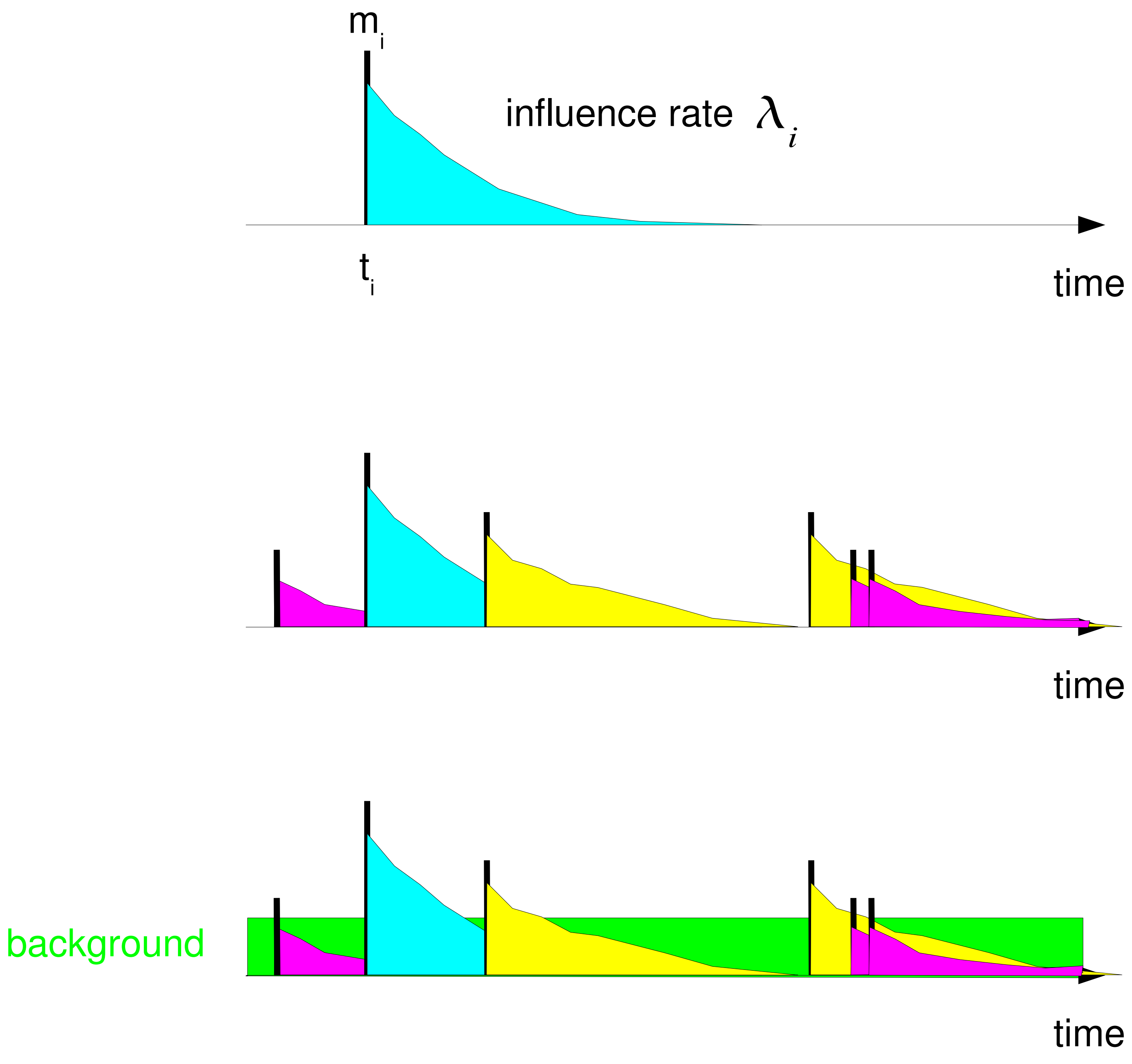
Seismicity rate $$\lambda(\underline{x},t)=\lambda_0+\sum_{t_i<t}\lambda_i(\underline{x},t)$$

background

linearity

Influence $$\lambda_i(\underline{x},t)=\lambda(|\underline{x}-\underline{x_i}|,t-t_i,m_i)$$

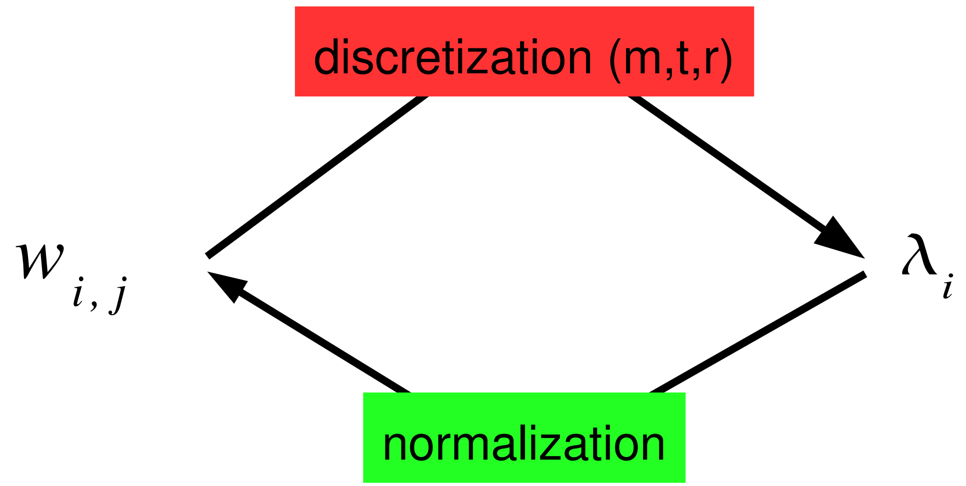
mean field


m_i
influence rate λ_i
t_i
time
time
background
time

$w_{i,j}$ **influence weight** = probability that earthquake #i triggered earthquake #j

$$\sum_{i=1}^{j-1} w_{i,j} + w_{0,j} = 1$$

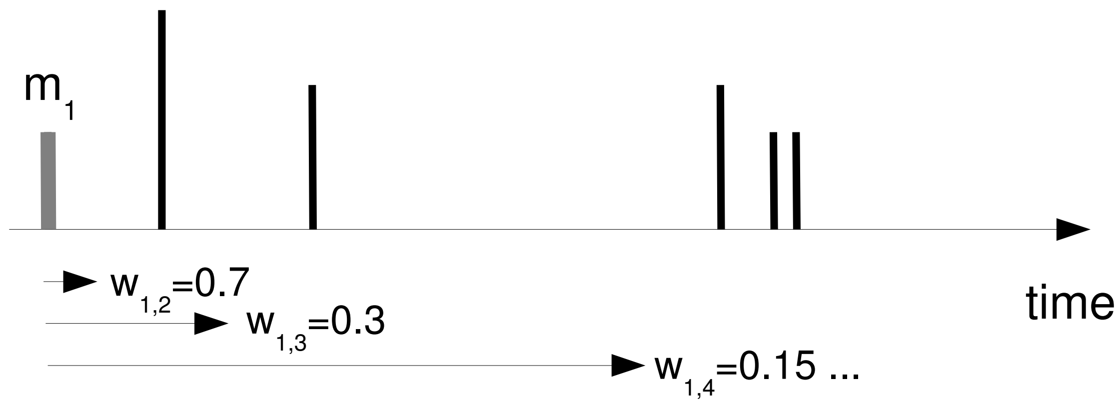
Algorithm



We start with **equal weights** for all earthquakes, and **iterate** the algorithm until the rates do not change with a new iteration (**convergence**).

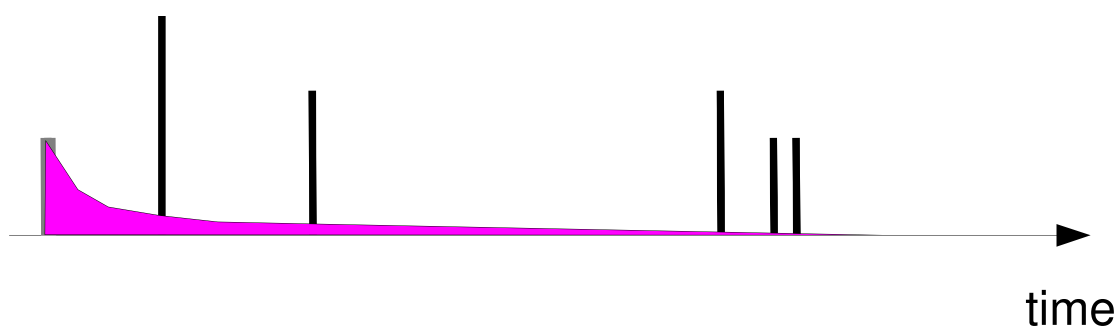
STEP 1

$$w_{i,j} \longrightarrow \lambda_i$$



Then **stacking** over all earthquakes with similar magnitudes:

$$\boxed{\lambda(r, t, m_1)}$$

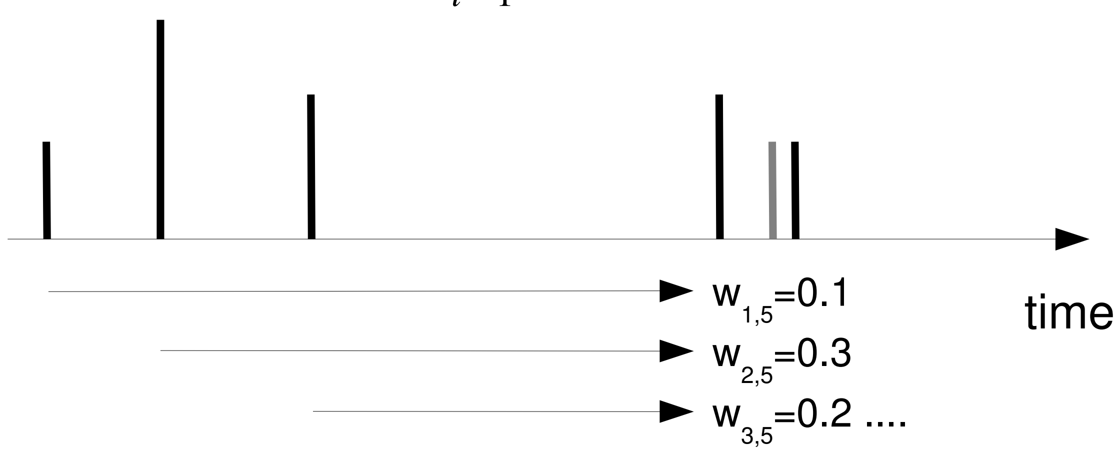


STEP 2

$$w_{i,j} \longleftarrow \lambda_i$$

$$\begin{cases} w_{i,j} \sim \lambda\left(\left|\underline{x}_j - \underline{x}_i\right|, t_j - t_i, m_i\right) \\ \\ w_{0,j} \sim \lambda_0 \end{cases}$$

+ normalization $\sum_{i=1}^{j-1} w_{i,j} + w_{0,j} = 1$



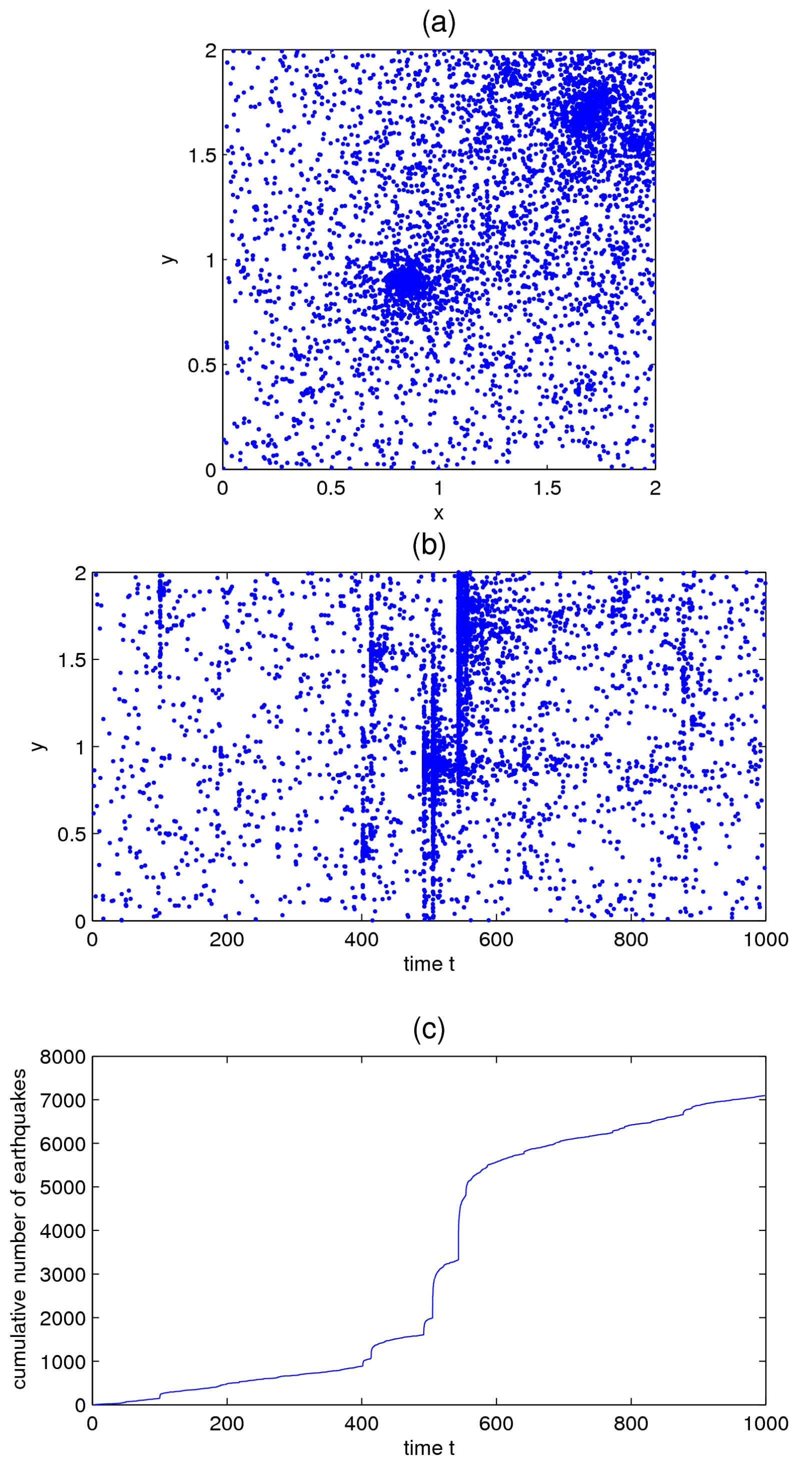
Test

ETAS model

N = 7102 earthquakes

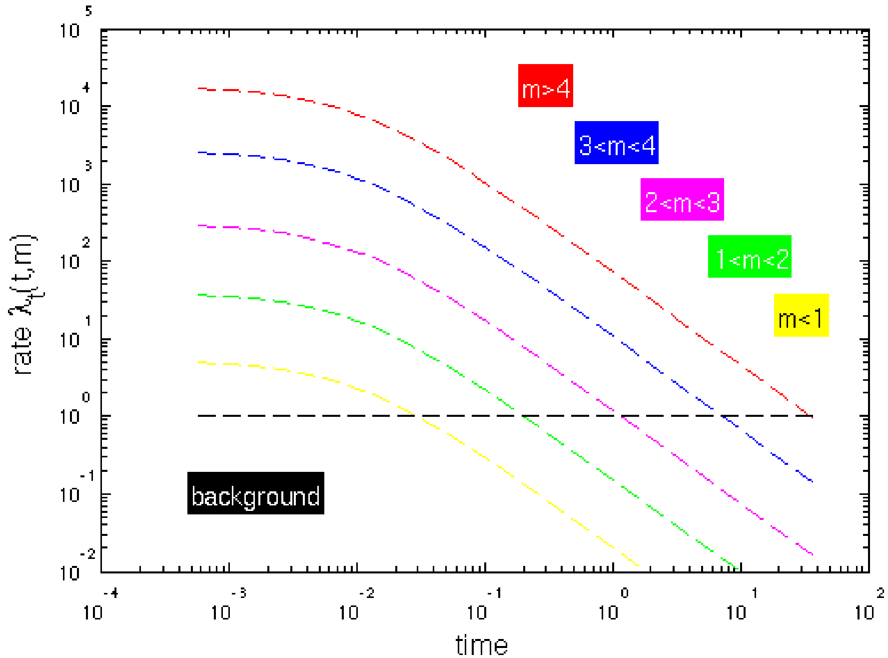
Magnitude range: 0 – 4.6

Branching ratio: 0.9

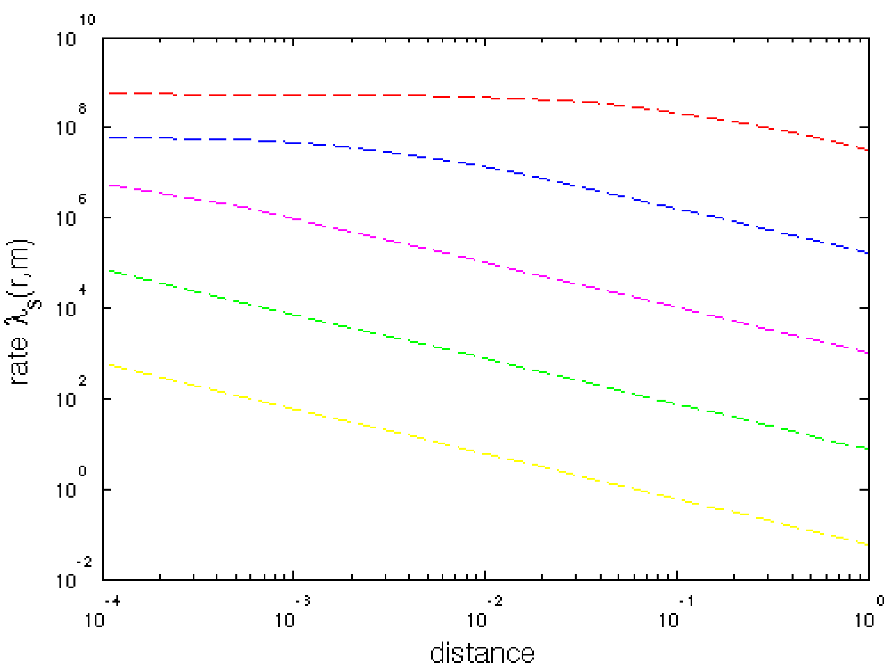


We decouple the spatial and the temporal dependence:

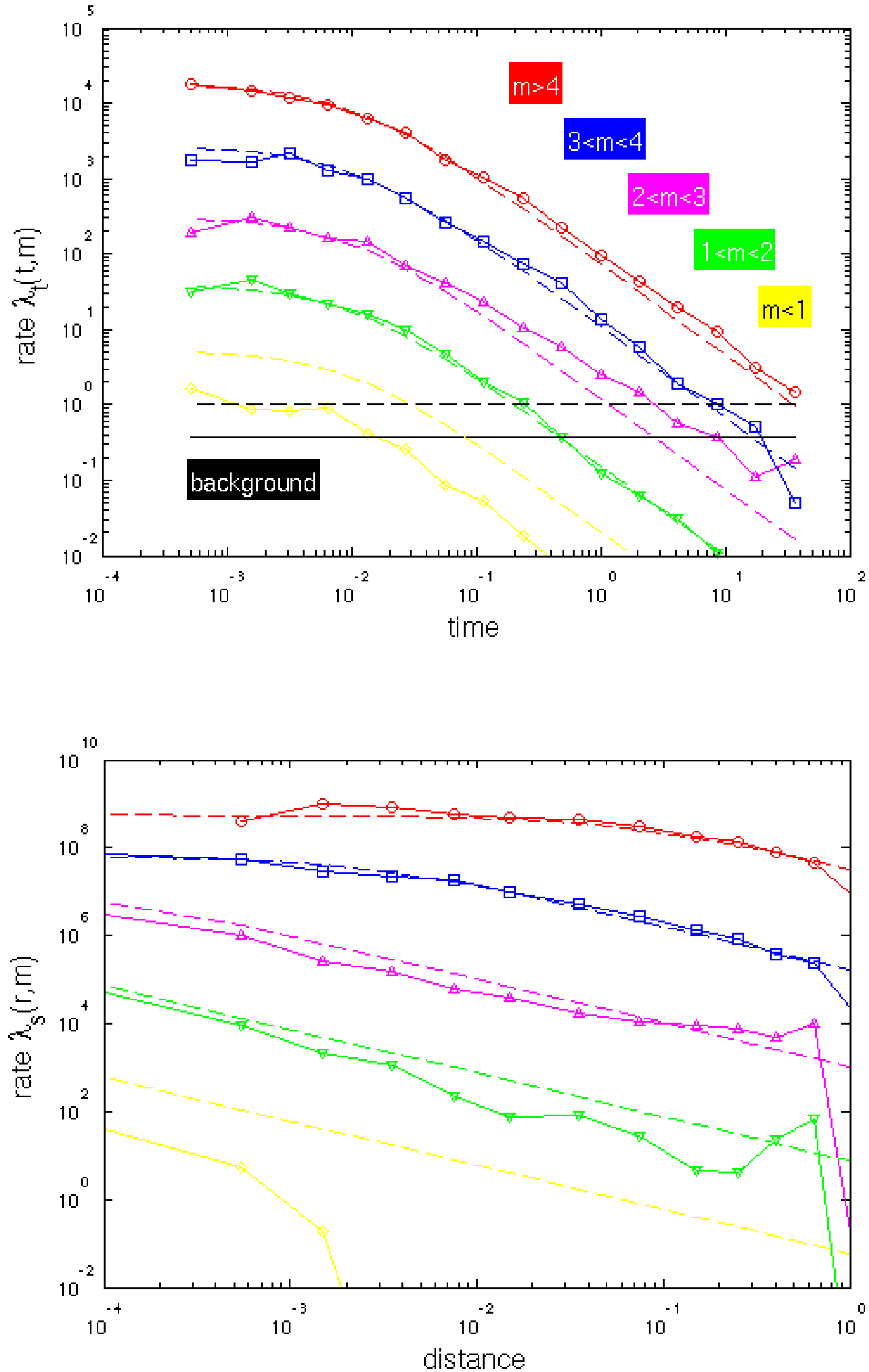
$$\lambda_i(\underline{x}, t) = \lambda_s(|\underline{x} - \underline{x}_i|, m_i) \times \lambda_t(t - t_i, m_i)$$



m>4
3<m<4
2<m<3
1<m<2
m<1
background
rate λt(t,m)
time



rate λs(r,m)
distance

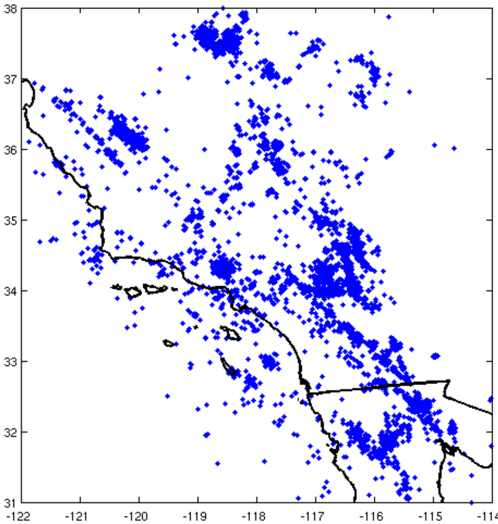


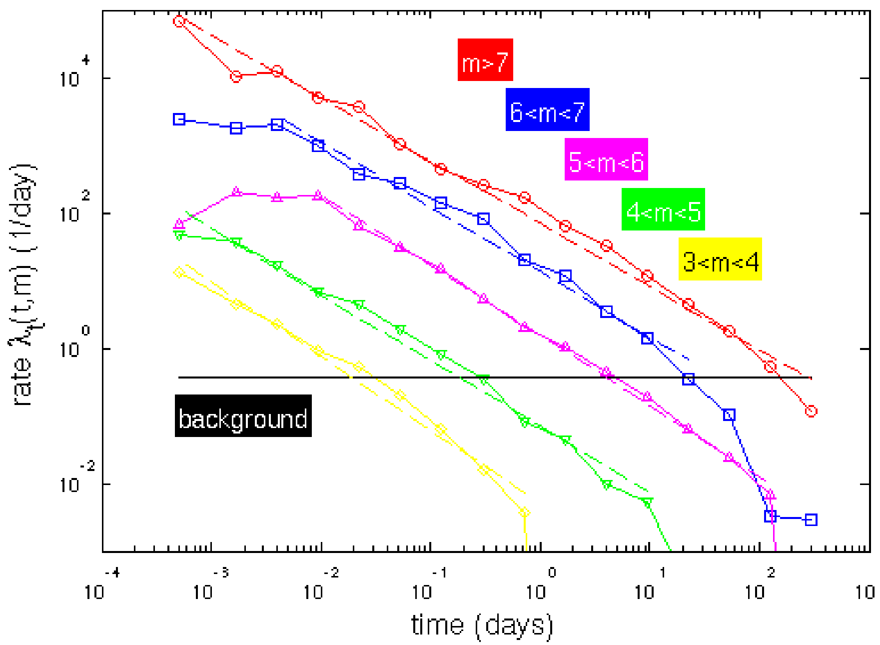
< 1' on a desktop computer

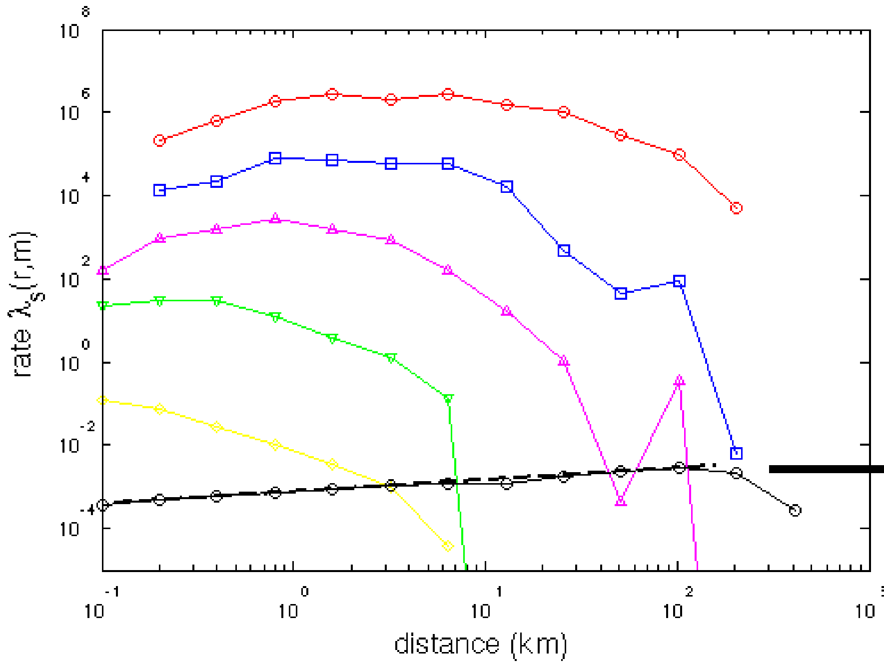
We analyse the earthquake dataset by Shearer et al. (2005)

- Central and southern California, 1984 - 2002
- M3+ earthquakes
- N = 6190 events

Background rate depends on distance to other « background earthquakes »



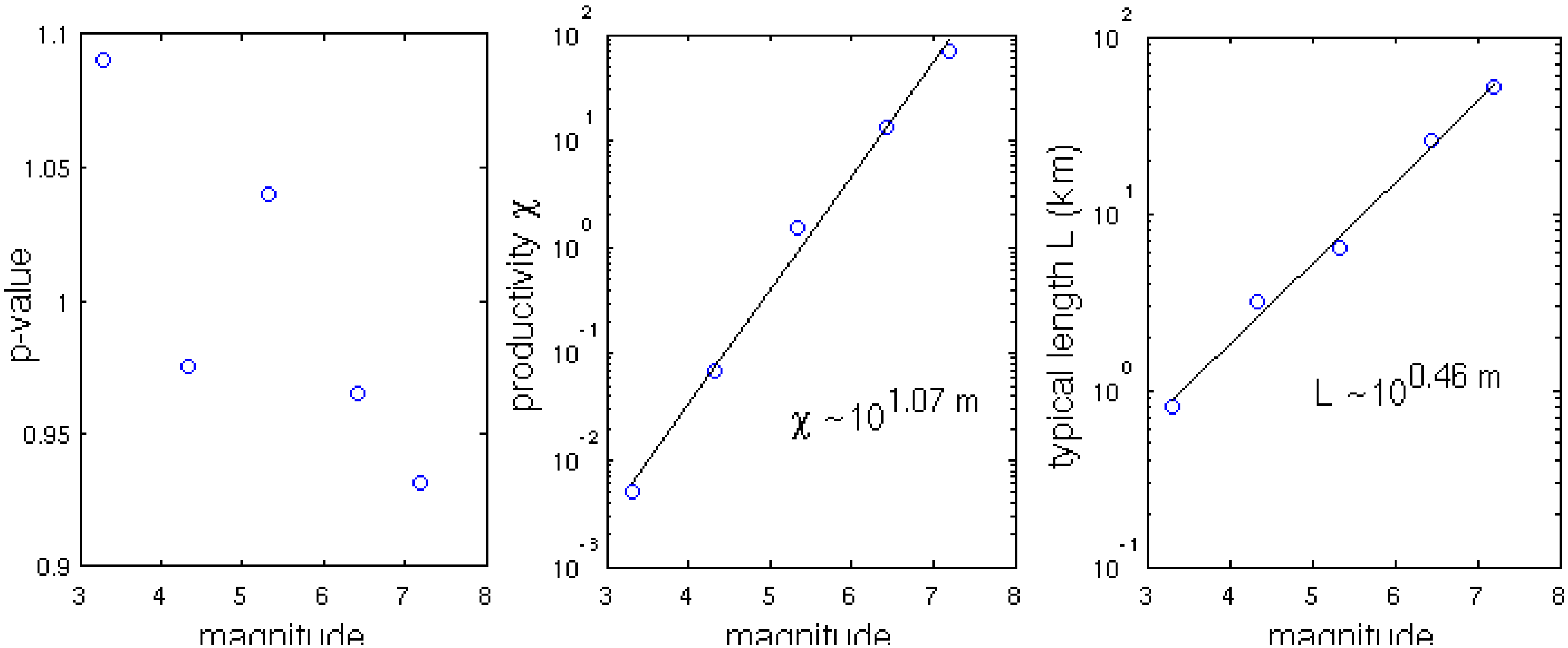


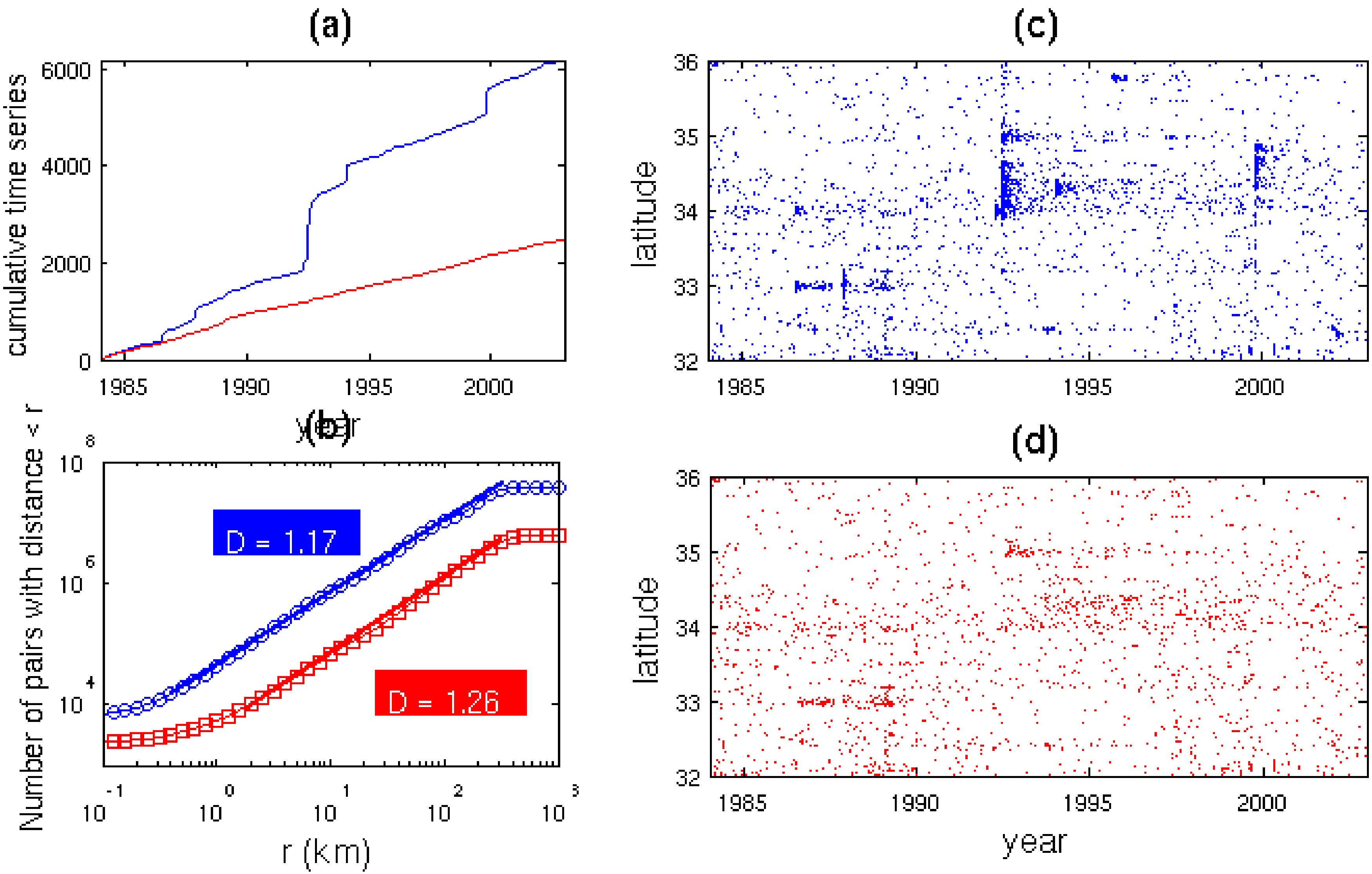


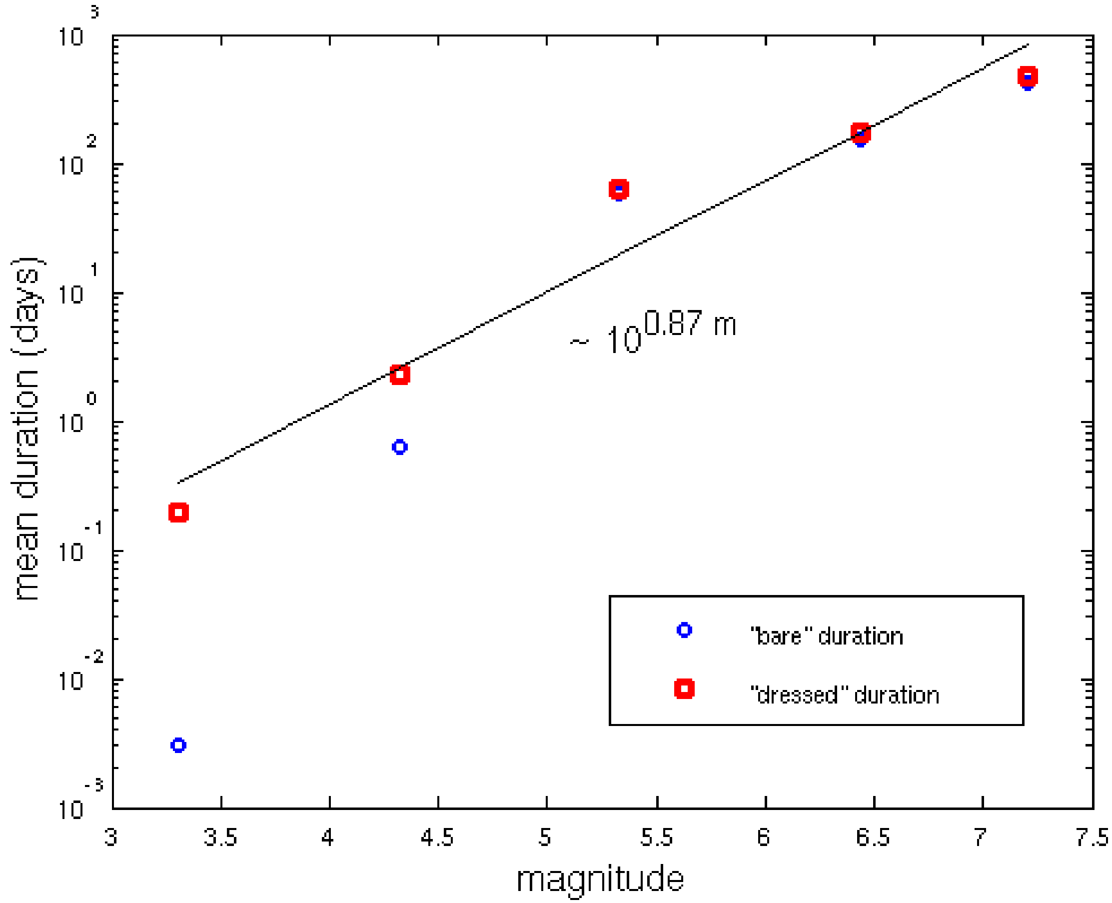
Fractal dimension

D = 1.28

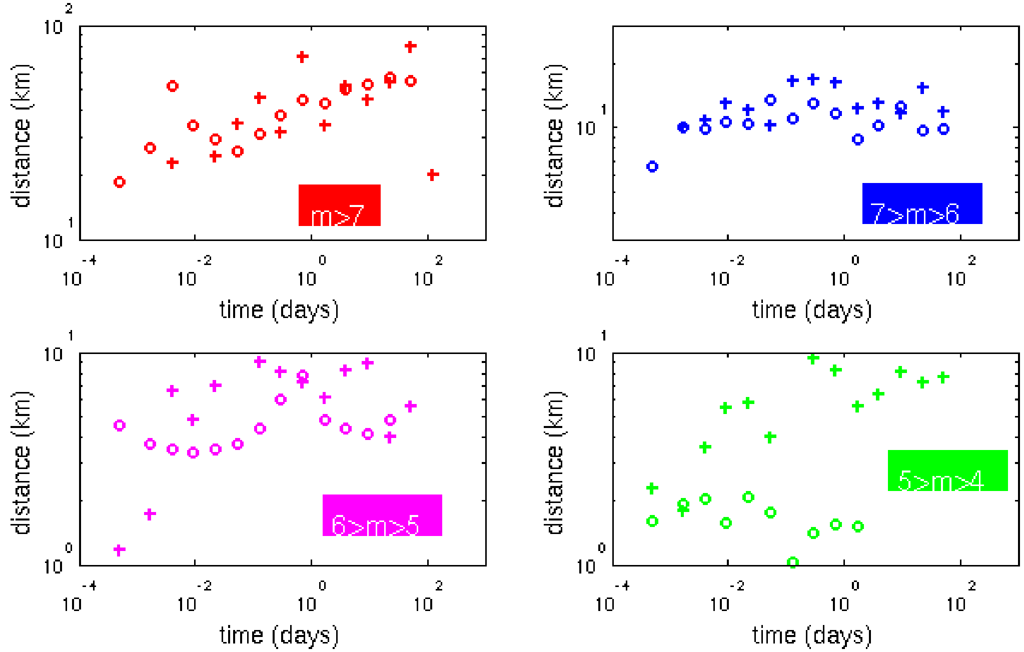
$$\lambda_t(t,m)=\chi \quad t^{-p}$$



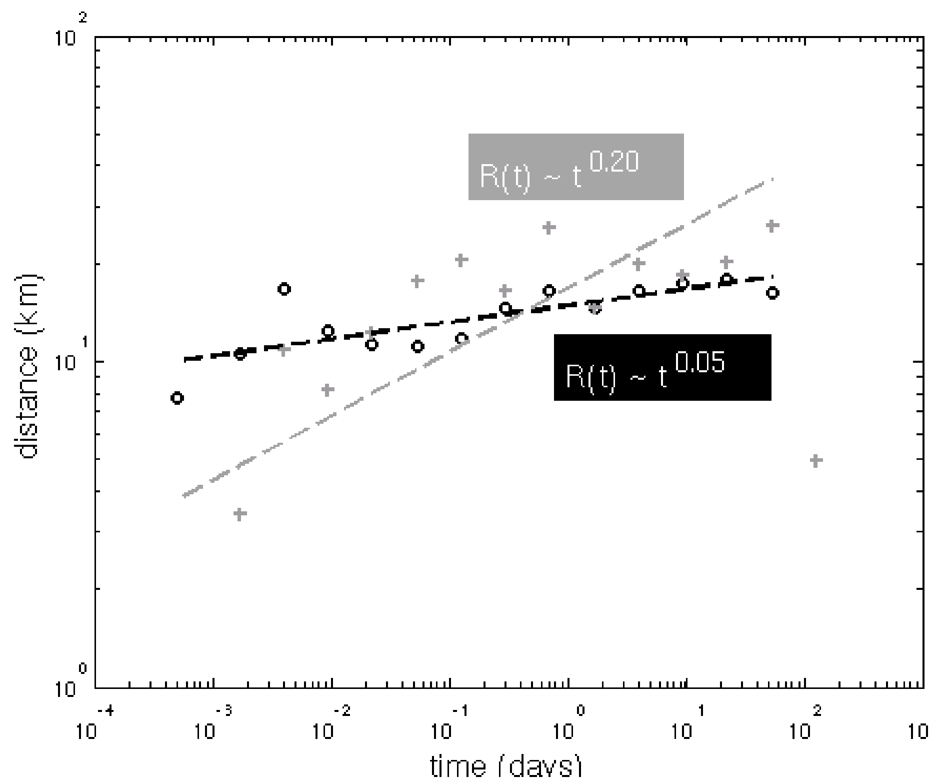

(a)
cumulative time series
6000
4000
2000
0
1985
1990
1995
2000
year
(b)
Number of pairs with distance < r
10^8
10^6
10^4
D = 1.17
D = 1.26
10^{-1}
10^0
10^1
10^2
10^3
r (km)
(c)
latitude
36
35
34
33
32
1985
1990
1995
2000
(d)
latitude
36
35
34
33
32
1985
1990
1995
2000
year



mean duration (days)
magnitude
~ 10^0.87 m
"bare" duration
"dressed" duration



o bare
+ dressed



Conclusions

* A simple algorithm can decipher the complex triggering patterns and estimate which earthquakes triggered what.

* It only relies on a small set of working hypotheses.

* It does not depend on a specific model (no parameters to invert for).

* It is simple to code and quick to run.